2-8 Polynomial Models in the Real World

Find a polynomial fuction whose graph passes through each set of points.

1. (7, 13), (10, −11), and (0, 4)

2. (−1, 9), (0, 6), (1, 5), and (2, 18)

For each set of data, what cubic model best fits the data? Use the model to estimate a value for the year 2012.

3. **U.S. Federal Spending**

Year	Total (billions $)
1965	630
1980	1,300
1995	1,950
2005	2,650

4. **World Population**

Year	Average Growth Rate (%)
1972	1.96
1982	1.73
1992	1.5
2002	1.22

For each set of data, compare two models and determine which one best fits the data. Which model seems more likely to represent each set of data over time?

5. **U.S. Homes**

Year	Average Sale Price (thousands $)
1990	149
1995	158
2000	207

6. **U.S. Crude Oil and Petroleum**

Month (2008)	Products Supplied (millions of barrels/day)
2	19.782
4	19.768
6	19.553

Use your models from Exercises 5 and 6 to make predictions.

7. Estimate the average sale price of homes sold in the United States for 1985, 1999, and 2020.

8. Estimate the number of barrels of crude oil and petroleum supplied per day for January, March, and October of 2008.

Do you know HOW?

Determine which type of model best fits each set of points.

9. $(-2, -1)$, $(0, 3)$, and $(2, 7)$

10. $(0, 3)$, $(3, 4)$, and $(5, 6)$

11. $(2, 3)$, $(4, 2)$, $(6, 4)$, and $(8, 5)$

12. $(-5, 6)$, $(-4, 3)$, $(0, 2)$, $(2, 4)$, and $(5, 10)$

13. **Vocabulary** Explain which form of estimation, interpolation or extrapolation, is more reliable.

14. **Reasoning** Is it possible to create a cubic function that passes through $(0, 0)$, $(-1, 1)$, $(-2, 2)$, and $(-3, 9)$? Explain.

15. **Writing** The R^2 value for a quartic model is 0.94561. The R^2 value for a cubic model of the same data is 0.99817. Which model seems to show a better fit? Explain.

Find a cubic and a quartic model for each set of values. Explain why one models the data better.

16.

x	−2	−1	0	2	3
y	−25	−4	3	23	40

17.

x	−2	−1	0	1	2
y	−65	−14	−4	2	90

Find a polynomial function whose graph passes through the points.

18. $(-14, 14)$, $(-10, 0)$, $(0, -1)$, $(8, 0)$, and $(12, 4)$

19. $(-3, -50)$, $(-2, -4)$, $(-1, 10)$, $(0, 7)$, and $(2, -23)$

20. **Think About a Plan** The table at the right shows the amount of carbon dioxide in the Earth's atmosphere for selected years. Predict the amount of carbon dioxide in the Earth's atmosphere in 2022. How confident are you in your prediction?

- How can you plot the data? (*Hint:* Let x equal the years after 1900.)
- What polynomial model should you use?

Year	CO^2 in atmosphere (ppm)
1968	324.14
1983	343.91
1998	367.68
2003	376.68
2008	385.60

SOURCE: The Weather Channel

Find a cubic model for each set of values. Then use the regression coefficient of each model to determine whether the model is a good fit.

21. $(-5, -60)$, $(-1, -5)$, $(0, 0.5)$, $(1, 8)$, $(5, 17)$, $(10, 32)$

22. $(8, -101)$, $(-1, 10)$, $(-8, 47)$, $(-10, 59)$

23. Air Travel The table shows the percent of on-time flights for selected years. Find a polynomial function to model the data. Use 1998 as Year 0.

Year	1998	2000	2002	2004	2006
On-Time Flights (%)	77.20	72.59	82.14	78.08	75.45

SOURCE: U.S. Bureau of Transportation Statistics

24. Writing Explain two ways to find a polynomial function to model a given set of data.

25. Error Analysis The table at the right shows the number of students enrolled in a high school personal finance course. A student says that a cubic model would best fit the data based on the $n + 1$ Point Principle. Explain why a quadratic model might be more appropriate.

Year	Number of Students Enrolled
2000	50
2004	65
2008	94
2010	110

26. Compare and Contrast The table shows the United States gross domestic product for selected years. Construct curves using cubic regression and quartic regression to model the data. Which curve seems most likely to model gross domestic product over the years?

Year	1960	1970	1980	1990	2000
GDP (billions $)	526.4	1038.5	2789.5	5803.1	9817.0

27. The table below shows the percentage of the U.S. labor force in unions for selected years between 1955 and 2005.

Year	1955	1960	1965	1970	1975	1980	1985	1990	1995	2000	2005
%	33.2	31.4	28.4	27.3	25.5	21.9	18.0	16.1	14.9	13.5	12.5

a. What is the average rate of change between 1955 and 1965? Between 1975 and 1985?

b. Make a scatter plot of the data. Which kind of polynomial model seems to be most appropriate?

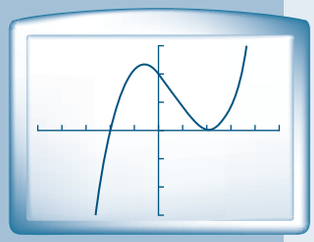
c. Use a graphing calculator to find the type of model from part (b).

d. Use the model you found in part (c) to predict the percent of the labor force in unions in the year 2020.

e. Reasoning Do you have much confidence in this prediction? Explain.

Challenge

28. Your friend's teacher showed the class a graph of a cubic polynomial in the ZDecimal window, which is $[-4.7, 4.7]$ by $[-3.1, 3.1]$. She then challenged the class to find the polynomial *without using cubic regression on their calculators*, and your friend succeeded. Follow your friend's steps and see if you can find the polynomial.



a. The graph resembles a parabola with vertex $(2, 0)$ near $x = 2$. Find the equation in standard form for that parabola.

b. Find the equation of a line in slope-intercept form through $(-2, 0)$ with slope 1. Multiply the linear expression by the quadratic expression from part (a) to get a cubic. (Leave it in factored form.) Graph the cubic function. What do you notice about the zeros and the y-intercept of the cubic function?

c. Multiply the cubic by a constant to change the y-intercept to 2. Graph the function to see if you've found the right polynomial. What is the function?

29. The graph at the right is that of a certain quartic polynomial in the ZDecimal window, which is $[-4.7, 4.7]$ by $[-3.1, 3.1]$. Find the equation of the quartic *without using quartic regression on your calculator*. You may leave it in factored form.