2-7 The Fundamental Theorem of Algebra

Name _____

Period_____

A Practice **Without using a calculator, find all the roots of each equation.**

1. $x^4 - 4x^3 + x^2 + 12x - 12 = 0$

2. $x^5 + 3x^3 - 4x = 0$

Find all the zeros of each function.

3. $y = x^4 - 6x^2 + 8$

4. $y = x^4 + 3x^3 + x^2 - 12x - 20$

Find the number of roots for each equation.

5. $5x^4 + 12x^3 - x^2 + 3x + 5 = 0$

6. $-x^{14} - x^8 - x + 7 = 0$

Find all the zeros for each function.

7. $y = x^3 - 5x^2 + 16x - 80$

8. $y = x^4 - 2x^3 + x^2 - 2x$

9. Vocabulary Given a polynomial equation of degree n, explain how you determine the number of roots of the equation.

10. Open-Ended Write a polynomial function of degree 4 with rational coefficients and two complex zeros of multiplicity 2.

11. Writing Describe when to use synthetic division and when to use the Quadratic Formula to determine the linear factors of a polynomial.

For each equation, state the number of complex roots, the possible number of real roots, and the possible rational roots.

12. $2x^4 - x^3 + 2x^2 + 5x - 26 = 0$

13. $x^5 - x^3 - 11x^2 + 9x + 18 = 0$

14. $-12 + x + 10x^2 + 3x^3 = 0$

15. $4x^6 - x^5 - 24 = 0$

Find all the zeros of each function. You may use a calculator and synthetic division to help you find one rational root.

16. $y = x^3 - 4x^2 + 9x - 36$

17. $f(x) = x^3 + 2x^2 - 5x - 10$

18. $y = 2x^3 - 3x^2 - 18x - 8$

19. $y = 3x^3 - 7x^2 - 14x + 24$

20. $g(x) = x^3 - 4x^2 - x + 22$

21. $y = x^3 - x^2 - 3x - 9$

22. $y = x^4 - x^3 - 5x^2 - x - 6$

23. $y = 2x^4 + 3x^3 - 17x^2 - 27x - 9$

24. Think About a Plan A polynomial function, $f(x) = x^4 - 5x^3 - 28x^2 + 188x - 240$, is used to model a new roller coaster section. The loading zone will be placed at one of the zeros. The function has a zero at 5. What are the possible locations for the loading zone?

- Can you determine how many zeros you need to find?
- How can you use polynomial division?
- What other methods can be helpful?

STEM **25. Bridges** A twist in a river can be modeled by the function $f(x) = \frac{1}{3}x^3 + \frac{1}{2}x^2 - x$, $-3 \le x \le 2$. A city wants to build a road that goes directly along the x-axis. How many bridges would it have to build?

26. Error Analysis Maurice says: "Every linear function has exactly one zero. It follows from the Fundamental Theorem of Algebra." Cheryl disagrees."What about the linear function y = 2?" she asks. "Its graph is a line, but it has no x-intercept." Whose reasoning is incorrect? Where is the flaw

Determine whether each of the following statement is *always*, *sometimes*, or *never* true.

27. A polynomial function with real coefficients has real zeros.

28. Polynomial functions with complex coefficients have one complex zero.

29. A polynomial function that does not intercept the x-axis has complex roots only.

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30. Reasoning A 4th degree polynomial function has zeros at 3 and $5 - i$. Can $4 + i$ also be a zero of the function? Explain your reasoning.

31. Open-Ended Write a polynomial function that has four possible rational zeros but no actual rational zeros.

32. Reasoning Show that the Fundamental Theorem of Algebra must be true for all quadratic polynomial functions.

Challenge

33. Use the Fundamental Theorem of Algebra and the Conjugate Root Theorem to show that any odd degree polynomial equation with real coefficients has at least one real root.

34. Reasoning What is the maximum number of points of intersection between the graphs of a quartic and a quintic polynomial function?

35. Reasoning What is the least possible degree of a polynomial with rational coefficients, leading coefficient 1, constant term 5, and zeros at $\sqrt{2}$ and $\sqrt{3}$? Show that such a polynomial has a rational zero and indicate this zero.

Find all roots given one root. Do not use a graphing calculator.

36) $3x^4 - 8x^3 + 2x^2 + 8x - 5 = 0$, x=1

37) $11x^4 + 100x^3 - 2x^2 - 100x - 9 = 0$ x = -9

38) $15x^4 - 44x^3 + x^2 - 14x + 6 = 0$ x=1/3

39) $12x^4 + 9x^3 + 8x^2 + 6x = 0$ x=-3/4

40) $7x^4 - 9x^3 - 5x^2 + 9x - 2 = 0$ x=1

41) $-15x^4 + 25x^3 + 64x^2 + 27x + 3 = 0$ x=-1

42) $x^3 - 7x^2 + 17x - 15 = 0$ x=3

Write the polynomial as the product of linear factors.

43) $f(x) = x^3 - 7x^2 + 9x + 9$

44) $f(x) = x^4 - 3x^3 - 8x^2 + 12x + 16$

45) $f(x) = x^3 - 3x^2 + 9x + 13$

46) $f(x) = x^4 - 17x^2 - 34x - 30$

ACTIVITY LAB

Exercises

Sketch a graph of each function. Check your answer using a graphing calculator.

1. $h(x) = (x + 6)(x - 7)$

2. $g(x) = (x + 1)(x - 3)(x - 5)$

3. $p(x) = x(x + 4)(x - 4)$

4. $h(x) = (x + 2)(x - 3)(x + 1)(x - 1)$

5. $f(x) = x^4 - 8x^2 + 16$

6. $k(x) = x^4 - 10x^2 + 9$

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